Global Navigation Satellite Systems (GNSS): Signal Processing and Navigation Techniques

Multiple Access and Spread Spectrum Ranging

Felix Antreich

Outline

Multiple Access Schemes

Spread Spectrum Signals

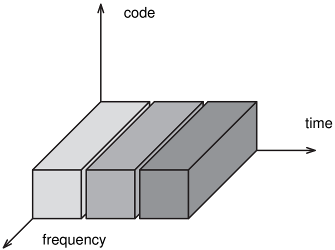
Processing Gain and Interference

Time-Delay Estimation

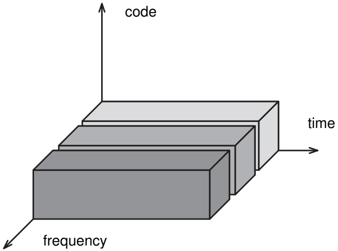
Multiple Access (MA)

- Several satellites need to share the same transmission medium and broadcast to the GNSS users to enable positioning
- The satellites need to share the transmission medium such that the GNSS users can separate the different satellites, perform ranging, and receive the navigation data
- The satellites need to share the available bandwidth by using channel access or multiple access (MA) techniques
- MA techniques have the aim to ensure that signals of the different satellites will be separated or even orthogonal
- In general there are three basic MA techniques:
  - Time division multiple access (TDMA)
  - Frequency division multiple access (FDMA)
  - Code division multiple access (CDMA)

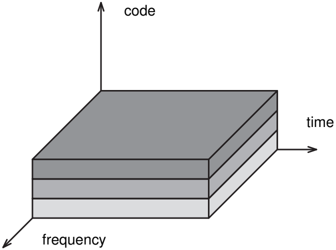
Different Basic MA Schemes



Time division multiple access (TDMA)



Frequency division multiple access (FDMA)



Code division multiple access (CDMA)

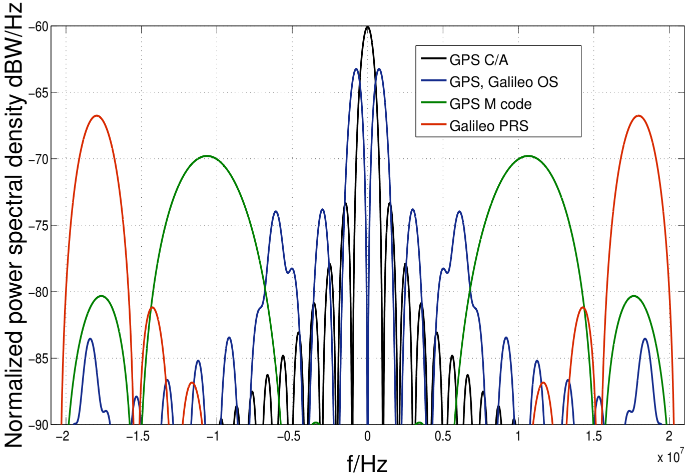
MA Schemes for GNSS (1)

- In principle each of these techniques can achieve the same aggregate spectral efficiency
  - Symbol rate per user of the channel (satellite)
  - Number of users per channel (satellites)
  - Sum symbol rate
- These basic MA techniques can also be combined to form hybrid combinations
  - Frequency division and time division (FD/TDMA)
  - Frequency division and code division (FD/CDMA)
  - ...
- For GNSS:
  - Number of in-view satellites (broadcast users) is quite low (around maximum 12 per system)
  - Data transmission demands (in general) do not play a prominent role
  - Mainly other performance measures have to be considered when choosing the appropriate MA technique

MA Schemes for GNSS (2)

- Signal design properties for GNSS:
  - Synchronization accuracy
  - Synchronization robustness
  - Inter system multiple access interference (MAI-R) or spectral separation
  - Intra system multiple access interference (MAI-A)
  - Interference robustness
  - Multipath performance
  - etc.
- Most GNSS (GPS, Galileo, Beidou) use direct sequence CDMA (DS-CDMA):
  - Each satellite uses a different code for transmitting its signal
  - Spectral separation between different GNSS in the same frequency band can be achieved by usage of different modulation schemes $\Rightarrow$ FD/CDMA
- GLONASS uses a CD/FDMA approach

FD/CDMA (DS-CDMA) for GNSS



L1/E1 1575.42 MHz

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Signal Model (1)

The received DS-CDMA or CD/FDMA baseband signal of one satellite is given as

$$x(t) = \sqrt{P}\, g(t-\tau)\, c(t-\tau) + n(t)$$

- P: Signal power
- $c(t)$: Pseudo random (PR) spreading sequence
- τ: Time-delay
- $g(t) \in \{-1, 1\}$: Binary navigation message data
- $n(t)$: White Gaussian noise with power spectral density $\frac{N_0}{2}$

Signal Model (2)

The PR sequence is

$$
\begin{aligned}
c(t) &= \sum_{m=-\infty}^{\infty} d_m \sqrt{T_c}\, \delta(t - mT_c) * p(t) \\
&= \sum_{m=-\infty}^{\infty} d_m \sqrt{T_c}\, p(t - mT_c)
\end{aligned}
$$

- $p(t)$: Chip pulse shape which is not necessarily restricted to be time-limited to only one chip interval
- T_c: Chip duration
- PR sequence with $\{d_m\} \in \{-1, 1\}$
- $T_d = N_d T_c$: PR sequence duration
- $N_d \in \mathbb{N}$: Number of chips of the PR sequence

Signal Model (3)

The PR sequence can be assumed (simplified) to be a zero mean

$$\mathrm{E}[d_m] = 0$$

and wide-sense cyclostationary (WSCS) sequence with

$$\begin{aligned} \mathrm{E}[d_m d_l^*] &= R_d[m, l-m] \\ R_d[m, l-m] &= R_d[m + pN_d, l-m], \quad p \in \mathbb{Z}. \end{aligned}$$

We also assume

$$\frac{1}{T_d} \int_{-\frac{T_d}{2}}^{\frac{T_d}{2}} c(t)c^*(t)dt = \int_{-\infty}^{\infty} \Phi_c(f)df = 1.$$

- $\Phi_c(f)$: power spectral density (PSD) of $c(t)$

Autocorrelation Function

The autocorrelation of $c(t)$ can be given as

$$\begin{aligned} R_c(\varepsilon) &= \frac{1}{T_d} \int_{-\frac{T_d}{2}}^{\frac{T_d}{2}} c(t)\; c^*(t+\varepsilon)\; dt \\ &= \int_{-\infty}^{\infty} |P(f)|^2\; \Phi_d(f)\; \mathrm{e}^{\mathrm{j}2\pi f\varepsilon}\; df \\ &= \int_{-\infty}^{\infty} |P(f)|^2\; \mathrm{e}^{\mathrm{j}2\pi f\varepsilon}\; df \end{aligned}$$

with

$$\int_{-\infty}^{\infty} |P(f)|^2\; df = 1.$$

- $P(f)$: Fourier transform of $p(t)$
- The WSCS sequence $\{d_m\}$ is not only pseudo random but random
- The power spectral density of the sequence $\{d_m\}$ is $\Phi_d(f) = 1$

Design of Autocorrelation and Cross-Correlation

Based on this interesting result we can conclude:

- The problem of optimizing cross-correlation and autocorrelation properties of the PR sequence $\{d_m\}$ can be treated separately as two different problems:
    1. Optimization of WSCS sequences and their properties
    2. Optimization of the chip pulse shape $p(t)$
- Easy analysis and understanding of the properties of chip pulse shapes $p(t)$ and PR sequences $\{d_m\}$

Correlation (1)

The receiver performs correlation with a period T_d using a replica signal based on a model. In case the time-delay τ is known to the receiver we can write

$$
\begin{aligned}
y[k] &= \frac{1}{T_d}\int_{\frac{T_d}{2}(2k-1)}^{\frac{T_d}{2}(2k+1)} \sqrt{P} g(t-\tau)c(t-\tau)c(t-\tau)dt \\
&+ \frac{1}{T_d}\int_{\frac{T_d}{2}(2k-1)}^{\frac{T_d}{2}(2k+1)} n(t)c(t-\tau)dt \\
&= \sqrt{P} g[k] + \check{n}[k]
\end{aligned}
$$

where $k = 0, 1, \ldots, K-1$ and $g[k] \in \{-1, 1\}$. The power of the signal and the noise after despreading can be given as

$$
\mathrm{E}\left[|\sqrt{P} g[k]|^2\right] = P
$$

$$
\mathrm{E}\left[|\check{n}[k]|^2\right] = \check{\sigma}_n^2 = \frac{N_0}{2T_d}.
$$

GALILEO INFORMATION CENTRE Brazil GALILEO EGNOS

Correlation (2)

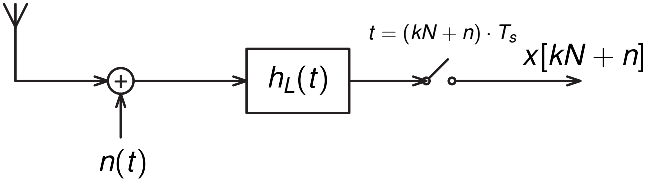
Considering the reception of N_{sat} satellites for either a DS-CDMA or CD/FDMA system we can write

$$y[k] = \frac{1}{T_d}\int_{\frac{T_d}{2}(2k-1)}^{\frac{T_d}{2}(2k+1)} \sqrt{P} g(t-\tau)c(t-\tau)c(t-\tau)dt + \underbrace{\sum_{i=1}^{N_{sat}-1} \frac{1}{T_d}\int_{\frac{T_d}{2}(2k-1)}^{\frac{T_d}{2}(2k+1)} \sqrt{P_i} g_i(t-\tau_i)c_i(t-\tau_i)c(t-\tau)dt}_{\approx 0} + \frac{1}{T_d}\int_{\frac{T_d}{2}(2k-1)}^{\frac{T_d}{2}(2k+1)} n(t)c(t-\tau)dt \approx \sqrt{P} g[k] + \breve{n}[k].$$

In case of DS-CDMA, the satellites are separated using different PR sequences while in case of CD/FDMA, the signals are separated spectrally (all satellites use the same PR sequence).

Discrete Signal Model (1)

In the receiver the observations are collected at N time instances in K periods, thus $x[kN+n] = x((kN+n)\,T_s)$ with $n = 0, 1, \ldots, N-1$ and $k = 0, 1, \ldots, K-1$, where $T_s = \frac{1}{2B}$ is the sampling duration. A simplified model of the received baseband signal after sampling can be given as



where

$$h_L(t) = 2\,B\frac{\sin(2\pi Bt)}{2\pi Bt}, \quad H_L(f) = \begin{cases} 1 & |f| \le B \\ 0 & \text{else} \end{cases}.$$

Discrete Signal Model (2)

Thus, with

$$T_d = N\,T_s = \frac{N}{2B}$$

we can write

$$\mathbf{x}[k] = \sqrt{P} g[k]\mathbf{c}[k;\tau] + \mathbf{n}[k],\ \ \mathbf{x}[k] \in \mathbb{R}^{N\times 1}$$

where

$$\begin{aligned}
\mathbf{x}[k] &= [x(kNT_s), \ldots, x((kN+n)\,T_s), \ldots, x((kN+N-1)\,T_s)]^{\mathrm{T}} \\
\mathbf{n}[k] &= [n(kNT_s), \ldots, n((kN+n)\,T_s), \ldots, n((kN+N-1)\,T_s)]^{\mathrm{T}} \\
\mathbf{c}[k;\tau] &= [c(kNT_s-\tau), \ldots, \ldots, c((kN+n)T_s-\tau), \ldots \\
&\qquad \ldots, c((kN+N-1)T_s-\tau)]^{\mathrm{T}}.
\end{aligned}$$

Discrete Signal Model (3)

We assume that

$$||\mathbf{c}[k;\tau]||_2^2 = N$$

while in general

$$||\mathbf{c}[k;\tau]||_2^2 \neq N, \forall\tau.$$

However, in many cases[1] we can assume that

$$||\mathbf{c}[k;\tau]||_2^2 \approx N, \forall\tau\ \forall k$$

if additionally $N \geq N_d$ and $N/N_d \in \mathbb{N}$ we get

$$\mathbf{c}[k;\tau] = \mathbf{c}(\tau), \forall k.$$

[1] e.g. in case of GPS C/A PR sequences with bandwidth $B \geq 1.023$ MHz.

Discrete Signal Model (4)

The autocorraltion function of the PR sequence can be given as

$$R_c[\varepsilon] = \frac{1}{N}\mathbf{c}^{\mathrm{T}}(0)\mathbf{c}(\varepsilon)$$

with

$$\frac{1}{N}\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{c}(\tau) = 1.$$

The receiver performs correlation with a period $N\,T_s$ using a replica signal based on a model. In case the time-delay τ is known and cross-correlation with other satellites can be neglected we can write

$$\begin{aligned} y[k] &= \frac{1}{N}\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{x}[k] = \frac{1}{N}\sqrt{P}g[k]\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{c}(\tau) + \frac{1}{N}\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{n}[k] \\ &= \sqrt{P}g[k] + \frac{1}{N}\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{n}[k]. \end{aligned}$$

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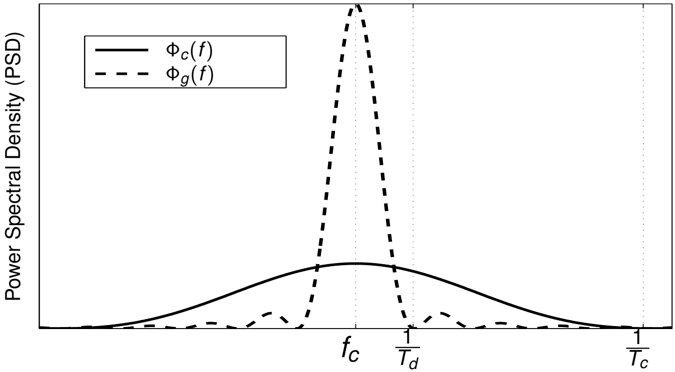
Spreading Factor (1)

The multiplication of the navigation data sequence $g(t)$ with the much faster oscillating PR sequence $c(t)$ introduces a spreading of the spectrum of the navigation data sequence of the factor

$$S = \frac{T_d}{T_c}$$

- S: Often called spreading factor
- In general $T_c << T_d$
- The spread signal thus can be “hidden” below the noise in case the maximum value of the PSD of the noise $\frac{N_0}{2}$ is larger than the maximum value of the PSD of the spread signal $\Phi_c(f)$

Spreading Factor (2)



- $\Phi_g(f)$: PSD of the navigation data sequence $g(t)$

Processing Gain (1)

The signal-to-noise ratio (SNR) before correlation can be given as

$$\mathrm{SNR_s} = \frac{\mathrm{E}\left[||\sqrt{P}g[k]\mathbf{c}(\tau)||_2^2\right]}{\mathrm{E}\left[||\mathbf{n}[k]||_2^2\right]} = \frac{P}{\sigma_n^2} = \frac{P}{N_0 B}.$$

In case the time-delay τ is known the SNR after despreading (correlation) can be given as

$$\mathrm{SNR_d} = \frac{\mathrm{E}\left[\left|\frac{1}{N}\sqrt{P}g[k]\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{c}(\tau)\right|^2\right]}{\mathrm{E}\left[\left|\frac{1}{N}\mathbf{c}^{\mathrm{T}}(\tau)\mathbf{n}[k]\right|^2\right]}$$

$$= \frac{P}{\frac{1}{N^2}\mathbf{c}^{\mathrm{T}}(\tau)\underbrace{\mathrm{E}\left[\mathbf{n}[k]\mathbf{n}^{\mathrm{H}}[k]\right]}_{=\sigma_n^2\mathbf{I}_N}\mathbf{c}^*(\tau)} = \frac{PN}{\sigma_n^2}$$

$$= \frac{PT_d 2B}{N_0 B} = \frac{P}{N_0 B_d} = \frac{P}{\breve{\sigma}_n^2}.$$

Processing Gain (2)

The relation between the SNR after and before despreading can be given by

$$\mathrm{SNR_d} = \frac{P}{N_0 B_d} = \frac{PB}{N_0 B_d B} = \frac{P}{N_0 B}\frac{B}{B_d} = \mathrm{SNR_s}\ G$$

where the so-called processing gain is

$$G = \frac{B}{B_d} = 2r\frac{T_d}{T_c} = 2r\,S,\ \ r \in \mathbb{R}^+.$$

- The processing gain G is a result of the change of the noise bandwidth and the resulting noise power
- The spreading factor S describes the spreading of $g(t)$ by multiplication with $c(t)$ and the processing gain G describes the gain in SNR after despreading

Interference

The signal-to-interference-plus-noise-ratio (SINR) before despreading in case of additional interference with power J (uniformly distributed across B) can be given as

$$\mathrm{SINR_s} = \frac{P}{BN_0 + J}$$

The SINR after dispreading is

$$\mathrm{SINR_d} = \frac{P}{B_d N_0 + \frac{B_d}{B} J} = \frac{P}{B_d N_0 + \frac{J}{G}}$$

- For broadband interference the SINR increase after despreading is dependent on the processing gain G
- System design can incorporate certain interference robustness
- For narrowband or partial band interference $c(t)$ spreads the jamming signal so that it appears as wideband Gaussian noise at the output of the correlator

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Maximum Likelihood Time-Delay Estimation (1)

For

$$\mathbf{x} = \mathbf{x}[0]$$

let us assume a random variable $\boldsymbol{x}$ has a multivariate Gaussian probability density function (pdf) parameterized by the parameter τ and thus we get

$$\mathrm{p}_{\boldsymbol{x}}(\mathbf{x};\tau) = \frac{1}{(2\pi\sigma_n^2)^{N/2}} \exp\left[-\frac{\|\mathbf{x} - \sqrt{P}\,\mathbf{c}(\tau)\|_2^2}{2\sigma_n^2}\right]$$

The likelihood function with respect to the parameter τ is given as

$$L(\mathbf{x};\tau) = \mathrm{p}_{\boldsymbol{x}}(\mathbf{x};\tau)$$

- $L(\mathbf{x};\tau)$ is a function of the parameter τ, which is to be estimated at a given realization of the random variable $\boldsymbol{x}$
- The pdf $\mathrm{p}_{\boldsymbol{x}}(\mathbf{x};\tau)$ is a function of the realization of the random variable $\boldsymbol{x}$ for a fixed value of the parameter τ

Maximum Likelihood Time-Delay Estimation (2)

Now the maximum likelihood estimator (MLE) can be given as

$$\hat{\tau} = \arg\max_{\tau} \{L(\mathbf{x};\tau)\} = \arg\max_{\tau} \{\log(L(\mathbf{x};\tau))\}.$$

The MLE is asymptotically (large N) unbiased and efficient. When further deriving the estimator we get

$$\begin{aligned}\hat{\tau} &= \arg\max_{\tau} \{\log(L(\mathbf{x};\tau))\} \\ &= \arg\max_{\tau} \left\{\log(1) - \log\left((2\pi\sigma_n^2)^{N/2}\right) - \frac{1}{2\sigma_n^2}\|\mathbf{x} - \sqrt{P}\mathbf{c}(\tau)\|_2^2\right\} \\ &= \arg\max_{\tau} \left\{-\|\mathbf{x}\|_2^2 + 2\sqrt{P}\,\mathbf{x}^{\mathrm{T}}\mathbf{c}(\tau) - P\,\|\mathbf{c}(\tau)\|_2^2\right\}\end{aligned}$$

As the first term does not depend on τ and the third term is constant with $\|\mathbf{c}(\tau)\|_2^2 \approx N, \forall_{\tau}$ as well as dropping the constant factor $2\sqrt{P}$ we can write

$$\hat{\tau} = \arg\max_{\tau} \{\mathbf{x}^{\mathrm{T}}\mathbf{c}(\tau)\} = \arg\max_{\tau} \{J(\tau)\}$$

Time-Delay Estimation with a Delay Locked Loop (DLL) (1)

In practice, time-delay estimation is performed using a DLL applying a gradient ascent method (step size $\mu > 0$) where the kth iteration can be given as

$$\hat{\tau}[k] = \hat{\tau}[k-1] + \mu \frac{\partial J(\hat{\tau}[k-1])}{\partial \tau}$$

The derivative can be approximated using the central difference quotient of length 2Δ,

$$\begin{aligned} \hat{\tau}[k] &= \hat{\tau}[k-1] + \frac{\mu}{2\Delta}\left(J(\hat{\tau}[k-1]+\Delta) - J(\hat{\tau}[k-1]-\Delta)\right) \\ &= \hat{\tau}[k-1] + \frac{\mu}{2\Delta}\left(\mathbf{x}^{\mathrm{T}}\mathbf{c}(\hat{\tau}[k-1]+\Delta) - \mathbf{x}^{\mathrm{T}}\mathbf{c}(\hat{\tau}[k-1]-\Delta)\right) \end{aligned}$$

A stochastic version (considering successive periods k) can be given as

$$\begin{aligned} \hat{\tau}[k] &= \hat{\tau}[k-1] + \frac{\mu}{2\Delta}\left(\mathbf{x}^{\mathrm{T}}[k-1]\mathbf{c}(\hat{\tau}[k-1]+\Delta)\right. \\ &\qquad \left. - \mathbf{x}^{\mathrm{T}}[k-1]\mathbf{c}(\hat{\tau}[k-1]-\Delta)\right) \end{aligned}$$

Time-Delay Estimation with a Delay Locked Loop (DLL) (2)

Without noise and assuming that the receiver uses signal matched correlators, the discriminator S-curve for a coherent early-late DLL can be given as

$$S(\varepsilon) = R_c(\varepsilon - \Delta) - R_c(\varepsilon + \Delta)$$
$$S[k;\varepsilon[k]] = R_c[\varepsilon[k] - \Delta] - R_c[\varepsilon[k] + \Delta].$$