Global Navigation Satellite Systems (GNSS): Signal Processing and Navigation Techniques

Signal Acquisition

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Outline

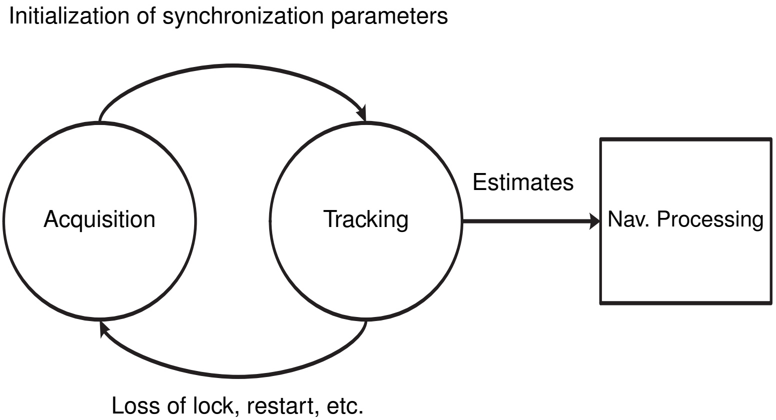
Introduction

Doppler and Time-Delay Estimation

Detection Problem

Parallel Time-Delay Acquisition

Overview


Initialization of synchronization parameters
Acquisition
Tracking
Estimates
Nav. Processing
Loss of lock, restart, etc.

Signal Acquisition

- Provides a rough estimate of the signal Doppler shift ν and time-delay τ
- Solves a maximum likelihood estimation problem in a coarse resolution
- Detects received satellites (separated by codes, DS-CDMA)
- For each satellite correlator outputs with the same time-delay and Doppler shift from different epochs/periods are used to form a decision variable
- If the decision variable passes a threshold, the signal (from a certain satellite) is assumed to be present/received
- Time-delay and Doppler shift estimates are used to initialize tracking loops

Outline

Discrete Signal Model (1)

Extending the discrete signal model by introducing a Doppler shift ν and a carrier phase ϕ we can write the complex baseband signal of one satellite as

$$\mathbf{x}[k] = \sqrt{P}\mathrm{e}^{\mathrm{j}\phi} g[k] \left(\mathbf{c}(\tau) \odot \mathbf{d}[k,\nu]\right) + \mathbf{n}[k] \;\in \mathbb{C}^{N\times 1}$$

where

$$\begin{aligned}
\mathbf{x}[k] &= \left[x(kN\,T_s), \ldots, x((kN+n)\,T_s), \ldots, x((kN+N-1)\,T_s)\right]^{\mathrm{T}} \\
\mathbf{n}[k] &= \left[n(kN\,T_s), \ldots, n((kN+n)\,T_s), \ldots, n((kN+N-1)\,T_s)\right]^{\mathrm{T}} \\
\mathbf{c}(\tau) &= \left[c(\tau), \ldots, \ldots, c(nT_s-\tau), \ldots, c((N-1)T_s-\tau)\right]^{\mathrm{T}} \\
\mathbf{d}[k;\nu] &= \left[\mathrm{e}^{\mathrm{j}2\pi\nu kNT_s}, \ldots, \mathrm{e}^{\mathrm{j}2\pi\nu\,(kN+n)\,T_s}, \ldots, \mathrm{e}^{\mathrm{j}2\pi\nu\,(kN+N-1)\,T_s}\right]^{\mathrm{T}}
\end{aligned}$$

as well as $\odot$ denoting the Hadamard-Schur product (element-wise multiplication), $n = 0, 1, \ldots, N-1$, $k = 0, 1, \ldots, K-1$, $N \geq N_d$, and $N/N_d \in \mathbb{N}$.

Discrete Signal Model (2)

We assume that the signal is filtered with an ideal lowpass filter $h_L(t)$ and subsequently sampled with a sampling frequency $f_s = \frac{1}{T_s} = 2B$. Thus, the noise $\mathbf{n}[k]$ is complex white Gaussian noise with

$$\begin{aligned} \mathrm{E}\left[\mathbf{n}[k]\right] &= \mathbf{0} \\ \mathrm{E}\left[||\mathbf{n}[k]||_2^2\right] &= N\sigma_n^2 \\ \mathrm{E}\left[\mathbf{n}[k]\mathbf{n}^{\mathrm{H}}[k]\right] &= \sigma_n^2\mathbf{I}_N \end{aligned}$$

We assume that quantization noise and thermal noise from the antenna and the LNA are included in $\mathbf{n}[k]$. Thus,

$$\sigma_n^2 = 2BN_0 + 2\sigma_q^2.$$

Maximum Likelihood Estimation (1)

For

$$\mathbf{x} = \mathbf{x}[0]$$

let us assume a random variable $\boldsymbol{x}$ has a multivariate Gaussian probability density function (pdf) parameterized by the parameters $\boldsymbol{\theta} = [\tau, \nu, \phi, P, g[0]]^{\mathrm{T}}$, and thus we get

$$\mathrm{p}_{\boldsymbol{x}}(\mathbf{x}; \boldsymbol{\theta}) = \frac{1}{(\pi\sigma_n^2)^N} \exp\left[-\frac{\|\mathbf{x} - \sqrt{P}\mathrm{e}^{\mathrm{j}\phi} g[0]\,(\mathbf{c}(\tau) \odot \mathbf{d}[0, \nu])\|_2^2}{\sigma_n^2}\right]$$

The likelihood function with respect to the parameter vector $\boldsymbol{\theta}$ is given as

$$L(\mathbf{x}; \boldsymbol{\theta}) = \mathrm{p}_{\boldsymbol{x}}(\mathbf{x}; \boldsymbol{\theta})$$

- $L(\mathbf{x}; \boldsymbol{\theta})$ is a function of the parameter vector $\boldsymbol{\theta}$, which is to be estimated at a given realization of the random variable $\boldsymbol{x}$
- The pdf $\mathrm{p}_{\boldsymbol{x}}(\mathbf{x}; \boldsymbol{\theta})$ is a function of the realization of the random variable $\boldsymbol{x}$ for a fixed value of $\boldsymbol{\theta}$

Maximum Likelihood Estimation (2)

Let us reparameterize the problem with

$$\alpha = \sqrt{P}\mathrm{e}^{\mathrm{j}\phi} g[0]$$

and

$$\boldsymbol{\theta} = [\tau, \nu, \alpha]^{\mathrm{T}}.$$

Now the maximum likelihood estimator (MLE) can be given as

$$\hat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \{L(\mathbf{x}; \boldsymbol{\theta})\} = \arg\max_{\boldsymbol{\theta}} \{\log\left(L(\mathbf{x}; \boldsymbol{\theta})\right)\}.$$

The MLE is asymptotically (large N) unbiased and efficient. When further deriving the estimator we get

$$\begin{aligned}\hat{\boldsymbol{\theta}} &= \arg\max_{\boldsymbol{\theta}} \left\{\log(1) - N\log(\pi\sigma_n^2) - \frac{1}{\sigma_n^2}\|\mathbf{x} - \alpha(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])\|_2^2\right\} \\ &= \arg\max_{\boldsymbol{\theta}} \{-\mathbf{x}^{\mathrm{H}}\mathbf{x} + \alpha^*(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}\mathbf{x} + \alpha\mathbf{x}^{\mathrm{H}}(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]) \\ &\quad -\alpha^*\alpha(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])\}.\end{aligned}$$

Maximum Likelihood Estimation (3)

Now, we can define the cost function

$$J(\boldsymbol{\theta}) = \alpha^*(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}\mathbf{x} + \alpha\mathbf{x}^{\mathrm{H}}(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]) \\ - \alpha^*\alpha(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]).$$

Taking the derivative of $J(\boldsymbol{\theta})$ with respect to α^* and equating to zero we get

$$\frac{\partial J(\boldsymbol{\theta})}{\partial \alpha^*} = (\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}\mathbf{x} \\ - \alpha(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]) = 0$$

and

$$\hat{\alpha} = \frac{(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}\mathbf{x}}{(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])} = \frac{(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu])^{\mathrm{H}}\mathbf{x}}{N}.$$

Maximum Likelihood Estimation (4)

Substituting $\hat{\alpha}$ in $J(\boldsymbol{\theta})$ with the above result we get

$$(\hat{\tau}, \hat{\nu}) = \arg\max_{\tau,\nu} \left\{ \left| \mathbf{x}^{\mathrm{H}} (\mathbf{c}(\tau) \odot \mathbf{d}[0, \nu]) \right|^2 \right\}.$$

In general such a problem can be solved by:

- Two-dimensional grid search
- Gradient method, e.g. Newton' s method (with "good" initialization)

The signal phase $\phi \pm \pi$ (considering $g[0] \in \{-1, 1\}$) and power P can also be determined using the estimate $\hat{\alpha}$ based on the estimates $\hat{\tau}$ and $\hat{\nu}$

$$\begin{aligned} \hat{\phi} \pm \pi &= \arg\{\hat{\alpha}\} \\ \hat{P} &= |\hat{\alpha}|^2. \end{aligned}$$

Grid Search

The final cost function that needs to be evaluated is also called cross ambiguity function (CAF) and for the period k can be given as

$$CAF[k;\tau,\nu] = |\mathbf{x}^{\mathrm{H}}[k](\mathbf{c}(\tau) \odot \mathbf{d}[k,\nu])|^2.$$

- To select a suitable strategy to evaluate the CAF and thus to find its maximum we have to inspect the shape of the CAF for the problem at hand
- Basic strategies in GNSS acquisition to evaluate the CAF are:
  - Serial search; pairs of time-delay and Doppler frequency values (bins) are evaluated one by one
  - Parallel time-delay acquisition; all possible time-delays are evaluated in parallel for each Doppler frequency bin
  - Parallel Doppler acquisition; all possible Doppler frequencies are evaluated in parallel for each time-delay bin

Outline

Satellite Detection

Signal acquisition determines the presence or absence of a satellite. Two hypothesis can be defined as

$$\begin{aligned} \mathcal{H}_1 &: \quad \mathbf{x}[k] = \alpha(\mathbf{c}(\tau) \odot \mathbf{d}[k,\nu]) + \mathbf{n}[k] \\ \mathcal{H}_0 &: \quad \mathbf{x}[k] = \mathbf{n}[k]. \end{aligned}$$

- $\mathcal{H}_0$, also called the null hypothesis, is fulfilled when the desired satellite signal is not received
- $\mathcal{H}_1$, also called alternative hypothesis, is fulfilled when the desired satellite signal is received
- This type of detection problem is called binary hypothesis testing

Likelihood Ratio Test (1)

Neyman-Pearson Lemma

To maximize the detection probability P_d for a given probability of false alarm $P_{fa} = \rho$ decide the hypothesis $\mathcal{H}_1$ if

$$LR(\mathbf{x}) = \frac{L(\mathbf{x};\boldsymbol{\theta},\mathcal{H}_1)}{L(\mathbf{x};\mathcal{H}_0)} > \gamma$$

where

$$\begin{aligned} P_{fa} &= \int_{\{\mathbf{x}|LR(\mathbf{x})>\gamma\}} L(\mathbf{x};\mathcal{H}_0)d\mathbf{x} = \rho \\ P_d &= \int_{\{\mathbf{x}|LR(\mathbf{x})>\gamma\}} L(\mathbf{x};\boldsymbol{\theta},\mathcal{H}_1)d\mathbf{x}. \end{aligned}$$

- $LR(\mathbf{x})$ is called likelihood ratio
- $L(\mathbf{x};\boldsymbol{\theta},\mathcal{H}_1)$ and $L(\mathbf{x};\mathcal{H}_0)$ denote the likelihood for hypothesis $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively

Likelihood Ratio Test (2)

Since

$$
\begin{aligned}
L(\mathbf{x};\boldsymbol{\theta},\mathcal{H}_1) &= \frac{1}{(\pi\sigma_n^2)^N}\exp\left[-\frac{\|\mathbf{x}-\alpha\left(\mathbf{c}(\tau)\odot\mathbf{d}[0,\nu]\right)\|_2^2}{\sigma_n^2}\right] \\
L(\mathbf{x};\mathcal{H}_0) &= \frac{1}{(\pi\sigma_n^2)^N}\exp\left[-\frac{\|\mathbf{x}\|_2^2}{\sigma_n^2}\right]
\end{aligned}
$$

the likelihood ratio can be given as

$$
LR(\mathbf{x}) = \exp\left[-\frac{1}{\sigma_n^2}\left(\|\mathbf{x}-\alpha\left(\mathbf{c}(\tau)\odot\mathbf{d}[0,\nu]\right)\|_2^2-\|\mathbf{x}\|_2^2\right)\right] > \gamma.
$$

Taking the logarithm on both sides we can write

$$
-\frac{1}{\sigma_n^2}\left(\|\mathbf{x}-\alpha\left(\mathbf{c}(\tau)\odot\mathbf{d}[0,\nu]\right)\|_2^2-\|\mathbf{x}\|_2^2\right) > \log(\gamma).
$$

Likelihood Ratio Test (3)

Rearranging we get

$$\begin{aligned}\alpha^* \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)^{\mathrm{H}} \mathbf{x} + \alpha \mathbf{x}^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right) & \\ -\alpha\alpha^* \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right) &> \sigma_n^2 \log(\gamma) \\ \alpha^* \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)^{\mathrm{H}} \mathbf{x} + \alpha \mathbf{x}^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right) &> \sigma_n^2 \log(\gamma) + PN \\ \mathrm{Re}\{\alpha \mathbf{x}^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)\} &> \frac{\sigma_n^2 \log(\gamma) + PN}{2}\end{aligned}$$

and defining a new threshold we get

$$\mathrm{Re}\{\alpha \mathbf{x}^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)\} > \gamma'.$$

Generalized Likelihood Ratio Test (GLRT) (1)

In case the parameters $\boldsymbol{\theta}$ are not known we can define the detector to decide $\mathcal{H}_1$ if

$$GLR(\mathbf{x}) = \frac{L(\mathbf{x}; \hat{\boldsymbol{\theta}}, \mathcal{H}_1)}{L(\mathbf{x}; \mathcal{H}_0)} > \gamma$$

where $\hat{\boldsymbol{\theta}}$ is the maximum likelihood estimate or $\hat{\boldsymbol{\theta}}$ maximizes $L(\mathbf{x}; \hat{\boldsymbol{\theta}}, \mathcal{H}_1)$. An equivalent form is to decide $\mathcal{H}_1$ if

$$GLR(\mathbf{x}) = \max_{\boldsymbol{\theta}} \left\{ \frac{L(\mathbf{x}; \boldsymbol{\theta}, \mathcal{H}_1)}{L(\mathbf{x}; \mathcal{H}_0)} \right\} = \max_{\boldsymbol{\theta}} \{ LR(\mathbf{x}) \} > \gamma.$$

Maximizing the the log-likelihood ratio we get

$$\max_{\boldsymbol{\theta}} \{ \log(L(\mathbf{x}; \boldsymbol{\theta}, \mathcal{H}_1)) - \log(L(\mathbf{x}; \mathcal{H}_0)) \} > \log(\gamma).$$

Generalized Likelihood Ratio Test (GLRT) (2)

Introducing the likelihoods of the two hypothesis we can write

$$\max_{\boldsymbol{\theta}} \Big\{ \alpha^* \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)^{\mathrm{H}} \mathbf{x} + \alpha \mathbf{x}^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)$$

$$-\alpha\alpha^* \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right) \Big\} \quad > \quad \sigma_n^2 \log(\gamma)$$

Taking the derivative of the cost function on the left hand side with respect to α^* and equate to zero we get

$$\hat{\alpha} = \frac{\left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)^{\mathrm{H}} \mathbf{x}}{N}.$$

Substituting α in the cost function above with $\hat{\alpha}$ we get

$$\max_{\tau,\nu} \{ |\mathbf{x}^{\mathrm{H}} \left(\mathbf{c}(\tau) \odot \mathbf{d}[0,\nu]\right)|^2 \} > N\sigma_n^2 \log(\gamma)$$

or

$$\max_{\tau,\nu} \{ CAF[k,\tau,\nu] \} > N\sigma_n^2 \log(\gamma) = \gamma'.$$

Outline

Discrete Fourier Transform (DFT)

First, we define the Discrete Fourier Transform (DFT) as

DFT

Analysis equation:

$$X[m] = \sum_{n=0}^{N-1} x[n]\mathrm{e}^{-\mathrm{j}2\pi mn/N} = \mathcal{DFT}\{x[n]\}.$$

Synthesis equation:

$$x[n] = \frac{1}{N}\sum_{m=0}^{N-1} X[m]\mathrm{e}^{\mathrm{j}2\pi mn/N} = \mathcal{DFT}^{-1}\{X[m]\}.$$

Circular Shift

In case of a finite sequence $x[n]$ a shift in time or in frequency only makes sense as a circular shift. The circular shift properties can be given as

$$\mathcal{DFT}\{x[(n-n_0) \bmod N]\} = \mathrm{e}^{-\mathrm{j}2\pi m n_0/N} X[m]$$

and

$$\mathcal{DFT}\{\mathrm{e}^{\mathrm{j}2\pi m_0 n/N} x[n]\} = X[(m-m_0) \bmod N].$$

The modulo operation can be defined as

$$a \bmod b = a - b\left\lfloor \frac{a}{b} \right\rfloor, \quad \lfloor x \rfloor = \max\{q \in \mathbb{Z} | q \leq x\}.$$

For example:

$$-4 \bmod 5 = -4 - 5\left\lfloor \frac{-4}{5} \right\rfloor = -4 - 5\lfloor -0.8 \rfloor = 1$$

and

$$4 \bmod 2 = 4 - 2\left\lfloor \frac{4}{2} \right\rfloor = 4 - 2\lfloor 2 \rfloor = 0.$$

Circular Cross-Correlation (1)

Consider two finite-duration sequences $x_1[n]$ and $x_2[n]$ of duration N with

$$\mathcal{DFT}\{x_1[n]\} = X_1[m]$$
$$\mathcal{DFT}\{x_2[n]\} = X_2[m].$$

The N-point circular cross-correlation can be defined as

$$z[n] = \sum_{p=0}^{N-1} x_1^*[p] x_2[(p+n) \bmod N], 0 \leq n \leq N.$$

The N-point DFT of $z[n]$ can be given as

$$Z[m] = \sum_{n=0}^{N-1} \sum_{p=0}^{N-1} x_1^*[p] x_2[(p+n) \bmod N] \, \mathrm{e}^{-\mathrm{j}2\pi mn/N}.$$

Circular Cross-Correlation (2)

We can further reformulate

$$
\begin{aligned}
Z[m] &= \sum_{n=0}^{N-1} \sum_{p=0}^{N-1} x_1^*[p] x_2[(p+n) \bmod N] \, \mathrm{e}^{-\mathrm{j}2\pi m(p+n)/N} \mathrm{e}^{\mathrm{j}2\pi mp/N} \\
&= \sum_{p=0}^{N-1} x_1^*[p] \mathrm{e}^{\mathrm{j}2\pi mp/N} \sum_{n=0}^{N-1} x_2[(p+n) \bmod N] \, \mathrm{e}^{-\mathrm{j}2\pi m(p+n)/N} \\
&= \sum_{p=0}^{N-1} x_1^*[p] \mathrm{e}^{\mathrm{j}2\pi mp/N} X_2[m] \mathrm{e}^{\mathrm{j}2\pi mp/N} \mathrm{e}^{-\mathrm{j}2\pi mp/N} \\
&= X_1^*[m] X_2[m].
\end{aligned}
$$

Thus, in practical problems it is convenient and quite efficient to derive the circular cross-correlation as

$$
z[n] = \mathcal{DFT}^{-1} \left\{ \left(\mathcal{DFT}\{x_1[n]\} \right)^* \cdot \mathcal{DFT}\{x_2[n]\} \right\}.
$$

Matrix Representation of the DFT (1)

The DFT matrix is an $N \times N$ symmetric matrix $\mathbf{W}_N$, where the m, nth element is given by

$$W_N^{mn} = \mathrm{e}^{-\mathrm{j}2\pi mn/N}$$

Thus, we can also write the DFT as

$$X[m] = \sum_{n=0}^{N-1} x[n] W_N^{mn}$$

and the inverse DFT (IDFT) as

$$x[n] = \frac{1}{N} \sum_{m=0}^{N-1} X[m] W_N^{-mn}.$$

The DFT of a vector $\mathbf{x} = [x[0], \ldots, x[n], \ldots, x[N-1]]^{\mathrm{T}}$ can be given as

$$\mathbf{x}_f = \mathbf{W}_N \mathbf{x}$$

Matrix Representation of the DFT (2)

Here, the vector in frequency is defined as

$$\mathbf{x}_f = [X[0], \ldots, X[m], \ldots, X[N-1]]^{\mathrm{T}}.$$

The IDFT can be given as

$$\mathbf{x} = \mathbf{W}_N^{-1}\mathbf{x}_f.$$

The following interesting properties of $\mathbf{W}_N$ exist:

$$\begin{aligned} \mathbf{W}_N^{-1} &= \frac{1}{N}\mathbf{W}_N^{*} \\ \mathbf{W}_N\mathbf{W}_N^{*} &= N\mathbf{I}_N \\ \mathbf{W}_N^{*} &= \mathbf{W}_N^{\mathrm{H}}. \end{aligned}$$

The DFT matrix $\mathbf{W}_N$ is a Vandermonde matrix.

Parallel Time-Delay Search (1)

We can now exploit the formulation of the circular cross-correlation using the DFT to perform a parallel time-delay search for each Doppler bin as

$$\mathbf{f}[k;\nu] = \boldsymbol{\xi}[k;\nu] \odot \boldsymbol{\xi}^*[k;\nu] = \begin{bmatrix} CAF[k;0,\nu] \\ CAF[k;T_s,\nu] \\ CAF[k;2T_s,\nu] \\ \vdots \\ CAF[k;NT_s,\nu] \end{bmatrix}$$

with

$$\boldsymbol{\xi}[k;\nu] = \mathbf{W}_N^{-1}\left[(\mathbf{W}_N\mathbf{x}[k])^* \odot (\mathbf{W}_N(\mathbf{c}(0) \odot \mathbf{d}[k,\nu]))\right]$$

where the Doppler bins and the evaluated time-delays are

$$\begin{aligned} \nu \in \mathcal{D}_\nu &= \{\nu_{min}, \nu_{min}+\Delta_\nu, \nu_{min}+2\Delta_\nu, \ldots, \nu_{max}-\Delta_\nu, \nu_{max}\} \\ \tau \in \mathcal{D}_\nu &= \{0, T_s, 2T_s, \ldots, NT_s\} \end{aligned}$$

and the Doppler resolution is Δ_ν.

Parallel Time-Delay Search (2)

The complete CAF can be given in a data matrix

$$\mathbf{F}[k] = [\mathbf{f}[k, \nu_{min}], \cdots, \mathbf{f}[k; \nu_{max}]]$$

Detection of available satellites can be performed by

$$F_{i,j} > N\sigma_n^2 \log \gamma = \gamma'$$

where $F_{i,j}$ is the element in the *i*th row and the *j*th column of **F**.

- For each PR sequence $\mathbf{c}(0)$ a matrix **F** has to be derived
- In case a satellite was detected using the GLRT the maximum element of **F** is used to derive the initial estimates of ν and τ
- The resolution for the Doppler has to be chosen such that the following parameter tracking process can be initialized properly

Parallel Time-Delay Search (3)